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B. Sc. Sem – II MJC-02

1. Sphere through a Circle

To find the equation of the sphere passing through a given circle.

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$
$$ax+by+cz+e=0$$

We know that the plane section of a sphere is a circle. So the equations

$$S \equiv x^2+y^2+z^2+2ux+2vy+2wz+d=0$$
$$P \equiv ax+by+cz+e=0$$

represent a circle

Now consider the equation

$S+KP=0$ that is

$$x^2+y^2+z^2+2ux+2vy+2wz+d+K(ax+by+cz+e)=0$$

or $x^2+y^2+z^2+(2u+Ka)x+(2v+Kb)y+(2w+Kc)z+(d+Ke)=0$ (1.1)

This represents a circle as (1) it is a second degree equations (1.1) are equal and (1.1) each of the coefficients of x^2, y^2, z^2 are equal and (1.1) each of the co-efficients of xy, yz, zx is zero.

Hence. $S+KP=0$ is the required equation of the spheres passing through the intersection of the circle $S=0$, $P=0$.

Que (2):— Find the sphere passing through the point (α, β, γ) and the circle $x^2+y^2+z^2=a^2$, $z=0$

Sol^n:— The given circle is

$$x^2+y^2+z^2=a^2, \quad z=0$$

The equation of any sphere through the given circle is $x^2+y^2+z^2-a^2+Kz=0$ —— (1)

Where K is arbitrary

If it passes through the point (α, β, γ).

then $\alpha^2+\beta^2+\gamma^2-a^2+K\gamma=0$

or $K = -\frac{1}{\gamma}(\alpha^2+\beta^2+\gamma^2-a^2)$

Substituting the value of K in (1)

We get

$$\gamma(x^2+y^2+z^2-a^2)-(\alpha^2+\beta^2+\gamma^2-a^2)z=0$$

This is the required equation of the sphere.

intersection of two spheres.

To prove that the curve of intersection of two spheres

$$S_1 \equiv x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$$

and $S_2 \equiv x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$

is a circle.

The Co-ordinates of a point common to the spheres (i.e on the curve of intersection) $S_1=0$ and $S_2=0$ that is also satisfy the equation $S_1-S_2=0$

$$2(u_1-u_2)x+2(v_1-v_2)y+2(w_1-w_2)z+d_1-d_2=0$$

which represents a plane. as it is a first degree equation in x, y and z

So the curve of intersection is on a plane. Also it is on any one of the sphere. So the curve of intersection is that of a plane and a sphere.

We know that the plane section of a sphere is a circle.

Hence. the two spheres intersect in a circle.